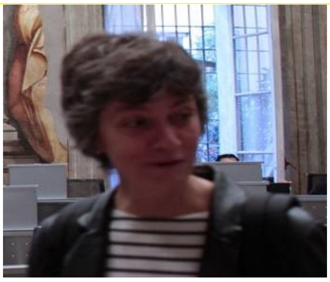

SNOW FLAKES, GRAMMARS AND GENERATIVE PROCESSES
Paper

Topics: Architecture

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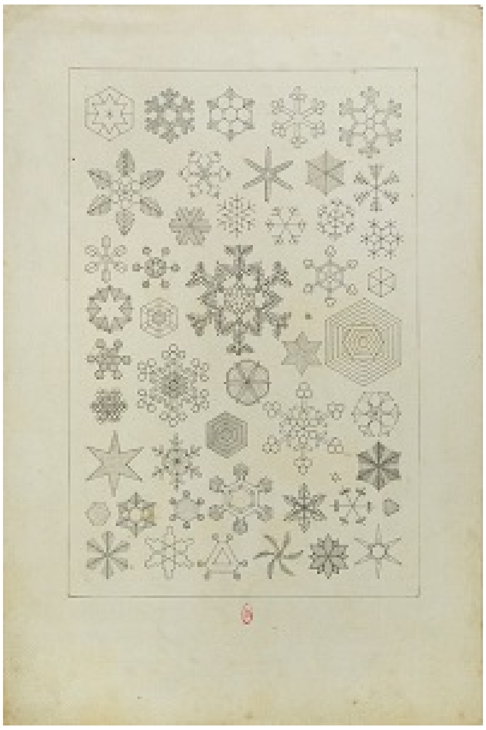
Abstract
Snow flakes have fascinated artists, scientists and philosophers (including most famous Kepler and Descartes), for a very long time, since at least 1250 (Albertus Magnus) in the Western World. The association of a strict hexagonal symmetry with such a diversity of specimens that any ice crystal may be considered unique is probably what fascinated all those people.
During the 19th century, there was a new approach to form among architects, which is well characterized by the term "grammar", as in the well known *Grammar of Ornament* (1856) by Owen Jones. Less widely acknowledged, French architect Jules Bourgoin (1838-1908) [1] wrote a *Grammaire élémentaire de l'ornement* in 1879. He is most known as a very accurate researcher in arabic art, but the topics he was interested in were very diverse, including snow flakes.
During the 20th century we encounter again the term "grammar" with the "shape grammars" introduced in 1971 by George Stiny and James Gips. One of the topics of this paper is to examine if shape grammars, and generative models in general, have presages in those previous grammars, especially regarding Bourgoin, which was the most thorough in trying to link his idea of grammar to philosophy, natural sciences, language, and mathematics.
Snow flakes are natural forms and examplary as such: they are obviously the result of a process of growth, which depends upon a few constraints, primarily the crystalline structure of ice, which is six-fold, and conditions of temperature and humidity, which can change dynamically in the micro-environment of the snow flake. They are then a good subject matter to confront to generative models, either cellular automata, the most obvious, or shape grammars or even fractal models. As such, they have at least an interest in an educational programme.
Beyond that, this paper uses snow flakes as an example of what "grammar" means for Bourgoin, and what it means now in the context of generative models. The topics of what "ornament" may mean for architecture nowadays is also addressed.



Ill: Jules Bourgoin. *«Étoiles de neige». Planche préparatoire pour un projet inédit de «Revue de l'ornement»*, (around 1877).

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Key words: snow flake, grammar, generative process

Main Reference:
[1] Maryse Bideault, Estelle Thibault, Mercedes Volait (dir.), *De l'Orient à la mathématique de l'ornement. Jules Bourgoin (1838-1908)*, InVisu, INHA, Éditions A. et J. Picard, Paris, 2015

Snowflakes, Grammars, and Generative Processes

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1. The quest for snowflakes
Snowflakes as forms, or shapes, are paradoxical. Though they are much less easy to observe directly than a lot of other natural forms, we are very familiar with them since our childhood. Actually, what we are familiar with is not natural snow flakes, but their iconic, or

symbolic, counterparts which, along with fir trees, reindeer, and so on, belong to Christmas folklore. All that folklore has much to do with the weather in northern Europe at the time of Christmas, and very little with that of Bethlehem, where Jesus' birth is supposed to have happened and to be celebrated by all those wintry symbols. The snowflake is somewhat mixed in the imagination linked to Christmas with the star that was supposed to have guided the three *magi* towards Jesus' manger, as a snowflake looks a little like a star, and is often referred to as "snow star".

Fig. 1: iconic snowflake, paper cut snow stars, ski school award

Iconic, or stylised, snowflakes, are certainly inspired by natural ones, but they retain only, as any icon, the main characters of snow crystals: six-fold radial symmetry, and dendritic (though not very detailed) arms. A main interest in stylised snowflakes is that you can make them in paper by folding a sheet according to the six-fold symmetry, and cutting out the "dendrites" with scissors, which is a good lesson in geometry for young children.

Observing actual snowflakes is not so easy. First, it must snow (and a "good" kind of snow), which is not something you can provoke, which occurs only in winter, and not so often in many regions. Then, you must observe *in situ*, you can hardly pick your snowflake and bring it home to comfortably observe it in a warmer place... As snowflakes are very small, you will also be helped in your observation if you are equipped with lenses or even better a microscope. And, finally, a single flake or rather a single crystal must deposit itself on a convenient background. You need some luck, as is related by two members of the Natural History Society of Belfast in 1838: "At Belfast, on the 14th of January 1838, about half an hour after noon, we remarked among some ordinary snow-flakes which, since the morning, had been falling very sparingly, some of the beautifully lamellar crystals which present so great a diversity of figure. We immediately hastened out of town, that we might have an opportunity of observing them undisturbed, and for about an hour enjoyed this high gratification. They then ceased to fall, the day became fine, and no return of the phenomenon took place." [1] We see here that snowflakes connoisseurs are not interested in the ordinary, but in so-called "lamellar" crystals, the remarkable ones, which we can then suppose are very few among snowflakes. In a note, one of the authors mentions the rarity of the phenomenon, not of snow itself, but of the *good* kind of snow.

Concerning their equipment, the authors add: "We were furnished only with the ordinary pocket lenses, and consequently were unable to attain that minute accuracy which is so desirable." [1] And finally, they mention how they could observe (and draw) single crystals: " (…) we most carefully sketched the crystals, either as they fell, or lay undisturbed on pieces of wood or metal exposed to the weather" [1].

Our two naturalists compare their figures (their sketches are unfortunately not printed in the magazine) with those described by Robert Hooke, who used the newly invented microscope to offer a great variety of natural forms, including snowflakes, in his famous *Micrographia* (1665), by the 18th Dutch physician John Nettis [2], who identified 91 specimens, and by the early 19th century Arctic explorer William Scoresby, who published in 1820 *An Account of the Arctic Regions*, in which he identified 96 distinct forms of remarkable snowflakes. What is at stake for our two snow watchers, is whether their figures are already present in the previous catalogues, or if they have discovered new specimens.

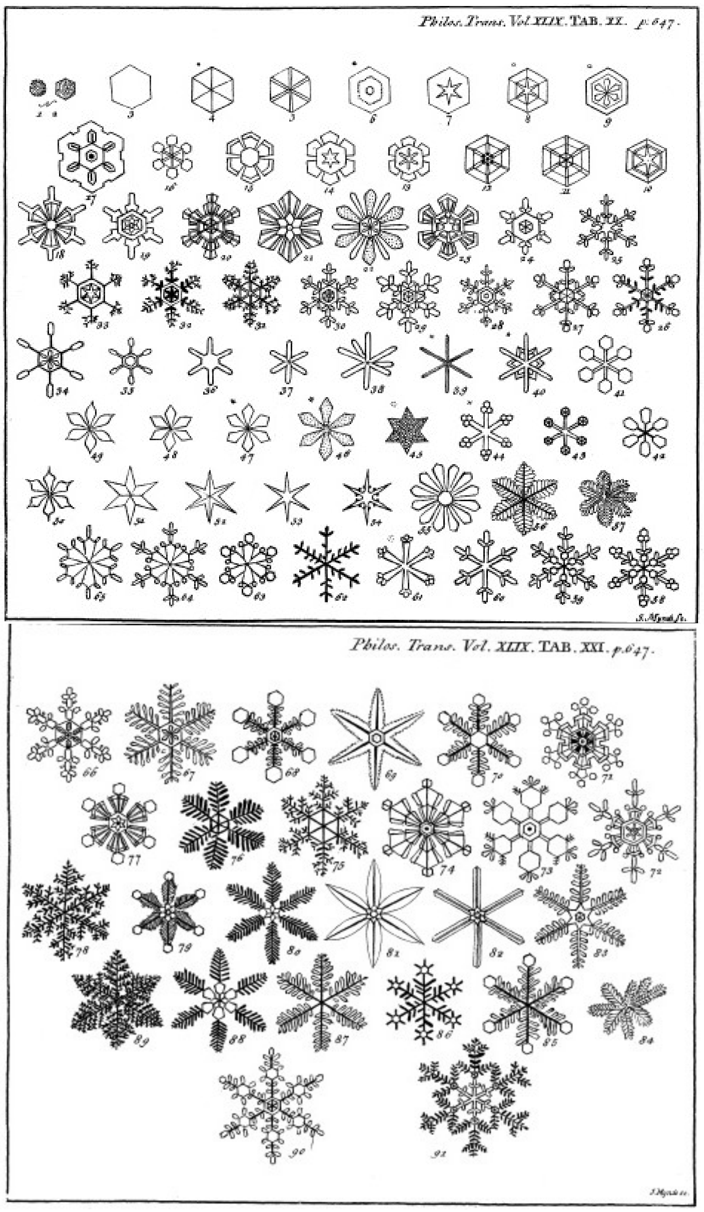

Fig. 2: "figures of snow" by John Nettis [2]

This attitude is typical of a time in which the concern is to inventory, and potentially classify, all the productions of nature. As well as for fauna and flora species, the thrill was to discover new "species" of snowflakes, as if inorganic forms could be treated as organic ones. Let us remind that Luke Howard used Linnean principles of natural classification to name categories of clouds ("Essay on modification of clouds", 1803) with the very names (*cumulus*, *stratus*, *cirrus*, ans their compounds) we still use nowadays.

Classifying and potentially naming the "species" of snowflakes was indeed a concern, and well into the 20th century, even if it is acknowledged by classifiers themselves that it is not an easy task. Kenneth G. Libbrecht [3] compares this attempt to that of the definition of breeds of dogs: "The definition of different breeds is decided upon by a committee of people, and really one can make up as many breeds as one wants. And no matter how many different breeds you define, some dogs will be mixed, not belonging to any one breed.” He made his own classification with 35 types, but other classifications exist, the first one having been made by Japanese physicist Ukichiro Nakaya, with 41 types (published in 1954), extended to 80 types by meteorologists C. Magono and C. W. Lee in 1966.

Fig. 3: Libbrecht’s classification of snowflakes [3]

Classifications were made easier by the invention of microphotography, at the very end of the 19th century, which could give more accurate accounts of the shapes of snowflakes. Wilson Bentley [4] was the first micro-photographer of snowflakes, he photographed his first snowflake on January 15, 1885 (he was twenty years old), and his photos are still a reference today, though a lot more have been taken since than (see [3]).

Fig. 4: some of Bentley’s micro-photographs of snowflakes

Classifying is not totally a “matter of taste”, as Libbrecht pretends it “somewhat” is. It is also an attempt to understand the atmospheric conditions that lead to such or such type of snowflake. Assumptions were made early, and our two Belfast “snow watchers” discuss, and contest, the assumption by Scoresby that “the configuration of the crystals “may be referred to the temperature of the air” (...)” [1], as they detect “several distinct figures falling simultaneously” which were, according to Scoresby, only produced at very distinct temperatures.

It is not before Nakaya was able to grow artificial snowflakes under controlled conditions in the 1930s that it became possible to refer snowflake shapes to temperature and humidity in the clouds. He produced the *Snow Crystal Morphology Diagram*, still in use nowadays.

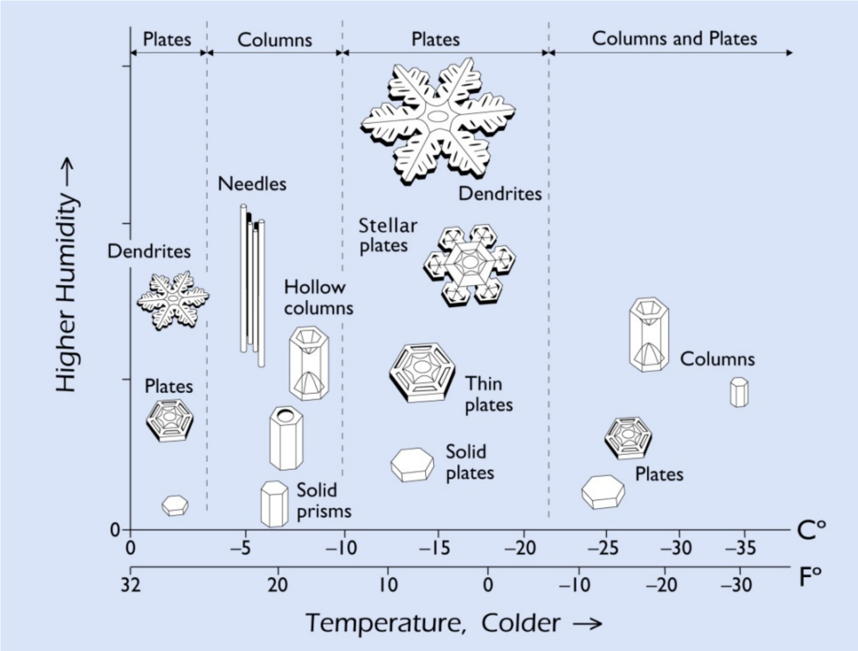

Fig. 5: snow crystal morphology diagram [3]

Nakaya was a precursor, but a lot of people now grow artificial snowflakes, and, what is more interesting, make videos of those growths, which are really amazing views [3]. Most of those experimental snowflakes grow lying on a plate, because it is difficult to let them be suspended in the air, and it would need too huge a tower to let them grow and fall like natural ones. But Norihiko Fukuta, a meteorologist, studied them in a vertical wind tunnel in 1988. Some were made in space, thus avoiding gravity, as in Space Shuttle Challenger during mission STS-8 in 1983, and in 2008, by remote control from Tsukuba Space Center of the Japanese Aerospace Exploration Agency in the Japanese Experiment module (Kibo) (see video here: [5]).

2. Snowflakes and generative processes

Having observed, classified, artificially grown, snowflakes, the next step is to algorithmically simulate them.

2.1 Hexagonal cellular automata

Hexagonal cellular automata are at first sight an obvious choice for mimicking what we are nowadays able to observe during the growth of an ice crystal. Such a crystal, as any crystal, obviously grows by aggregation, and the symmetry of the crystal reflects that of the molecule of ice, so using a hexagonal lattice makes sense. And, indeed Stephen Wolfram himself allocates a few pages to the growth of crystals, starting with snowflakes [6, pp.369-374].

Hexagonal cellular automata are not much more difficult to implement than their orthogonal counterparts. You can even use a classical 2D array and just have to deal with the coordinates of the neighbours, taking into account the fact that cells are shifted every other row. As with other 2D cellular automata the number of possible rules is overwhelming. Each cell has 6 adjacent neighbours, and the number of configurations is in all $2^7 = 128$. So there are potentially 2^{128} (about 3.4×10^{38}) different rules. One can reduce this number for reasons of symmetry, and only retain 13 distinct configurations for the neighbours, but it nevertheless leads to 2^{26}, or 67 108 864 different rules. So one generally considers totalistic cellular automata, where you count the number of neighbours, not bothering with their locations. One cell can have 0 to 6 neighbours, and its fate, whether it is occupied or not, is to be occupied, or not, at the next stage. This leads to 2^{14} or 16 384 possible rules: almost manageable!

You can yet reduce the number mentioned previously by considering only strictly growing cellular automata, i. e. that no cell will disappear once it is set (as one can imagine that an iced molecule will not melt), and that no cell appears with no neighbour. You then have 2^6, that is 64, possible results, which is very manageable, and one can easily see all the results. Most are not interesting because they either abort or develop into a filled hexagon (though one must notice that some snowflakes are such). Below are some of those that develop into intricate figures:

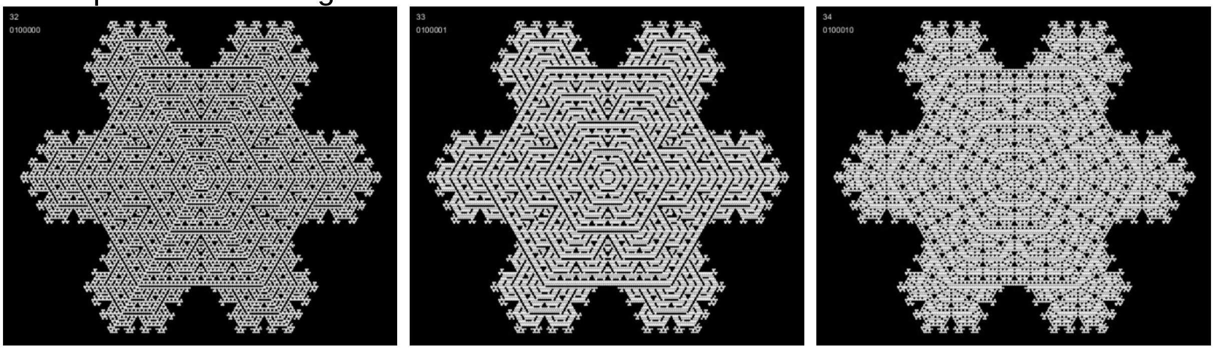

Fig. 6: some results of 2-state strictly growing CA

One must say that they are a little disappointing... Even if Wolfram claims that “with models based on simple programs as cellular automata, there is no problem in dealing with more complicated shapes, and indeed, as we have seen, it is actually quite easy to reproduce the basic features of the overall behaviour that occurs in real snowflakes” [6, p. 372]. The problem is that the dendrites, so typical of real snowflakes, do not appear, or, when they begin, they are very soon absorbed by the growth of the figure.

And yet Wolfram had well stated the facts: “At a basic level, one knows that snowflakes are formed when water vapour in a cloud freezes into ice, and that the structure of a given snowflake is determined by the temperature and humidity of the environment in which it grows, and the length of time it spends there. (…) The growth inhibition (…) is a result of the fact that when water freezes into ice, it releases a certain amount of latent heat [6, p. 372]. Those facts were actually mentioned (and questioned) by our two naturalists about their snowy walk outside Belfast. It has actually been sanctioned by more recent experiments.

One must acknowledge that “boolean” cellular automata (when cells are either occupied or not), i. e. discrete 2-state ones, and which consider the immediate neighbourhood, are not able to generate the dendritic quality of snowflakes. Happily for us, Angela M. Coxe and Clifford A. Reiter, from Lafayette College, Pennsylvania, have gone farther, and invented a more sophisticated cellular automaton, with two improvements [7]. First, they considered what they call “fuzzy” states, that is, continuous ones, ranging from 0 to 1. Discrete states are deduced from them, a cell “freezing” when its fuzzy state exceeds 0.5. Secondly, they do not consider only the 6 adjacent cells, but the 12 next level ones, that are of two types, those at the tips of the hexagon, and those between. They have elaborated a somewhat complex formula according to the number of each type of neighbours, and to their fuzzy values. At the start, there is a single frozen cell against a background where every cell has got the same fuzzy value. With the same formula, changing the value of the background leads to dramatic changes in the result of the growth. One must say that their work was worth the effort, because we can clearly see those dendritic formations (at least during the process of growth) which escaped the more classical model.

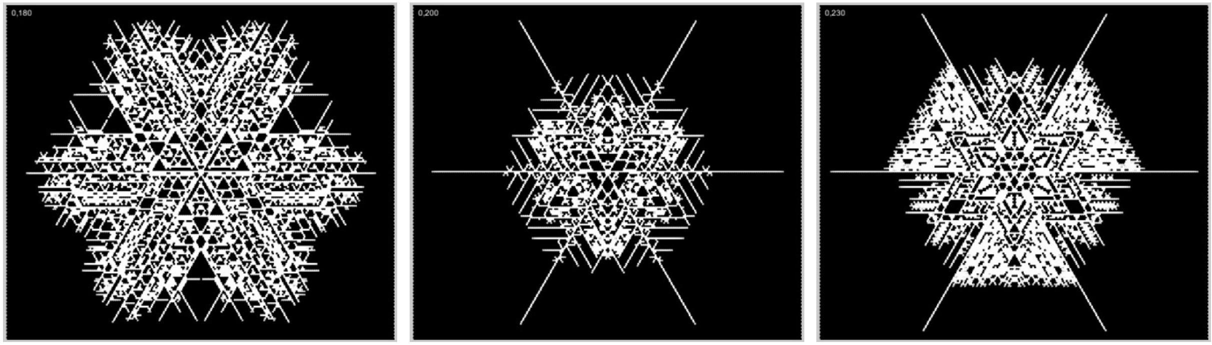

Fig. 7: some results of fuzzy hexagonal CA

One can wonder if it would be possible to make a more simple model, anyway...

2.2 Fractal models

In a very different attitude from cellular automata, we may examine the snowflake figures themselves, in their configuration. The hexagonal symmetry is a given, but can we detect some inner symmetry, i. e. some self-similar or fractal characteristics? Some of the figures or photographies of real snowflakes may lead to that hypothesis.

A simple IFS (Iterated Function System) [8] can be implemented. Let us remind that the IFS theory claims that any fractal, or self-similar figure, is the attractor of a system of transformations, the only constraint being that they must be “contracting”, and that it suffices to determine these transformations to obtain the figure itself. The six-fold symmetry being the strongest feature of snowflakes, the simplest IFS one can think of comprises 6 affine transformations, consisting of the rotations and translations that afford this symmetry, composed with a scaling in x and another one in y, the ratio of which we shall be able to modify.

IFS may be actually implemented in two ways: either one starts with some set of points, on which are applied all the transformations, and those transformed points are transformed again, and so on (Deterministic Algorithm); or one starts with one point, on which is applied one of the transformations chosen at random, and again on the result, and so on (Random Iteration Algorithm, or “Chaos Game”). The end result is the same, which relies on what an attractor is.

Some results are interesting to compare with snowflakes. Here the Random Iteration Algorithm is used, and pixels may be coloured in two ways: either by the colour affected to the transformation that was used to get to it, which clearly shows the set of transformations of the IFS, or by a level of grey linked to the number of times that particular pixel was hit, which shows the intricate structure of the fractal.

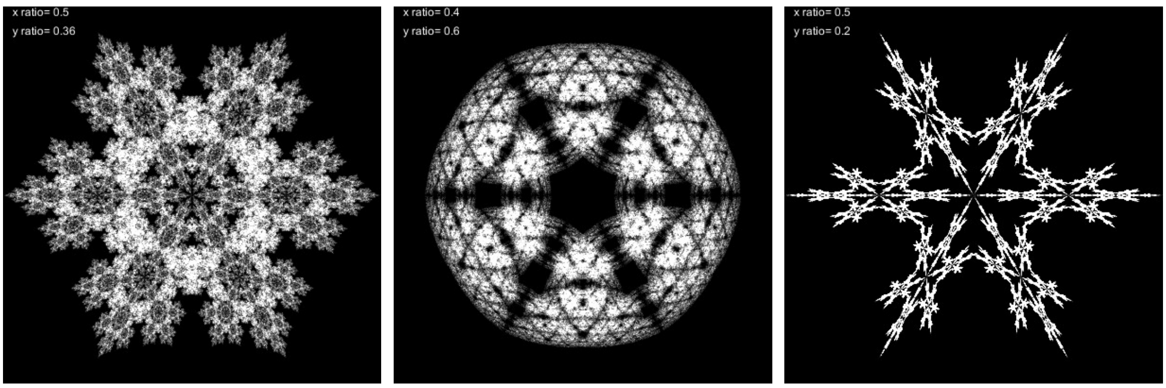

Fig. 8: some results from symmetrical 6-transformations IFS

A second attempt has been made, with an IFS where a simple scaling at the centre, with no rotation nor translation, is added to the previous transformations.

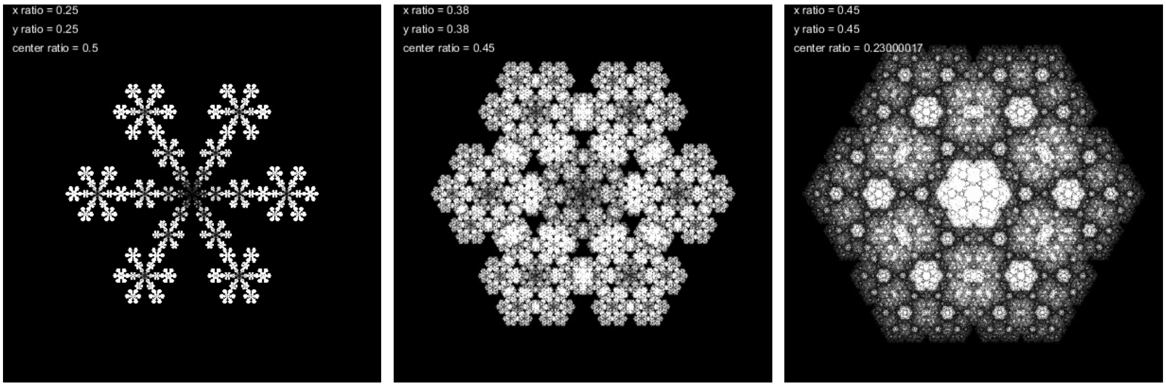

Fig. 9: some results from symmetrical 7-transformations IFS

Though results are nice, they lack the dendritic characteristics of some natural snowflakes. In order to obtain this quality, we must renounce to a whole self-similarity, and arrange 6 identical fractals around a centre. This unique fractal is easy to be made dendritic, as it is one of the stereotypical fractals, illustrated by Barnsley's famous fern.

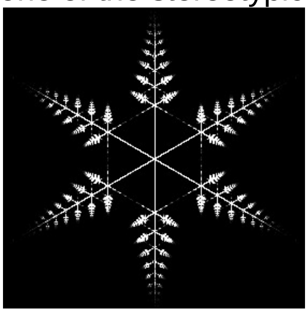

Fig. 10: result of a composite of 6 dendritic IFS

3. From grammars to generative processes

This part is more an agenda than an account of results, more a list of questions and hypotheses than firm conclusions. It reflects the start of a research program about potential links between "grammars" as they were understood in the 19th century, and "shape grammars" developed in the 20th century. The interest marked by Jules Bourgoin, himself such an "ornament grammarian", in snowflakes, was a pretext to begin that exploration.

There is some sort of parallelism between the way naturalists observed, described, and classified fauna, flora, and even inorganic species like snowflakes, and the attitude that prevailed in the 19th century towards human productions, and specifically ornament. Owen Jones' *Grammar of ornament* is at first sight an inventory of ornaments classified by civilisations, they are "specimens" in a kind of ornament *herbarium*. This analogy is reinforced by the means used to exhibit specimens in both cases: drawing.

Obviously, the word "grammar" does not refer to natural history, but to linguistics, or what was rather called *philology* at the time, i. e. the science of language. But there is a part of that science which also consists in collecting, describing and classifying the different varieties of human language, and which may be linked to natural history.

Jules Bourgoin (1838-1908), himself author of a *Grammaire élémentaire de l'ornement* in 1879, was certainly not a naturalist. He was interested in human productions, and insisted that they were "pure invention", independent of any natural model [9, p. 217]. He warned against "those links one makes too fast between creations of nature and human works". He even went against Viollet-le-Duc, the mentor of the time, who proposed, in Bourgoin's

Arts Arabes introduction, a parallel (judged fallacious by Bourgoin) between "the microscopic aspect of some organic tissues of plants or animals and the marvellous embroideries of Oriental ornament." [9, p. 218]. Let us remark that Viollet-le-Duc seemed to like mixed metaphors...

However, if Bourgoin most probably did not go and observe plants or even rocks, for instance, in the field, and even less snowflakes (taking moreover in account that he was very often in Egypt or such countries), he was interested, according to his reading notes [9, pp. 314-318], in phytomorphology, crystallography, and, very especially in snowflakes: fourteen references, among them those already cited by Hooke, Nettis, and Scoresby.

Let us make a first hypothesis here. Though Bourgoin was contemporary of the invention of photography, and was photographed himself in 1883, all references about snowflakes he consults date from before this invention and consequently rely on drawings. Those drawings, not only were of carefully chosen and so-called remarkable snowflakes, but idealised them, stylised them, as all drawings do. We can then assume that Bourgoin was not so much interested in snowflakes for themselves as by their graphic power, by the games with six-fold symmetry those drawings suggest. His preparatory plates of "snow stars", which were destined to a "Revue de l'ornement" which was finally not published, were issued from sketch notes he took from Nettis, Scoresby and other scientific sources, which show the precise geometrical analysis he made of those figures, and the attention he brought to the way they were classified.

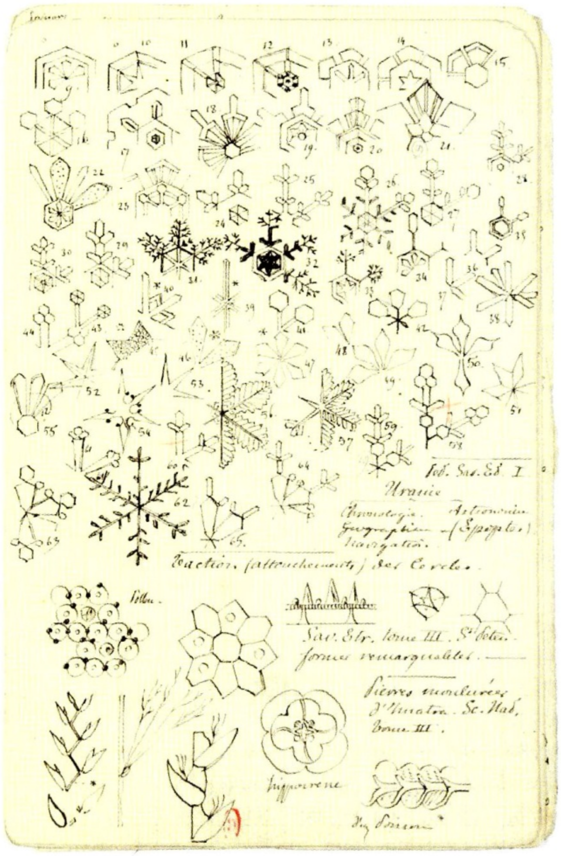

Fig. 11: Bourgoin's notebook: sketches and notes taken from Nettis

But in the preparatory plates, we see extrapolations from those references, as for instance a figure with curved branches which is unlikely to be found among true snowflakes. The figures are not arranged according to any scientific classification of snowflakes, some are exact transcriptions of Nettis' figures (one may compare fig. 12 with fig. 2), but some others seem to be variations on hexagonal symmetry.

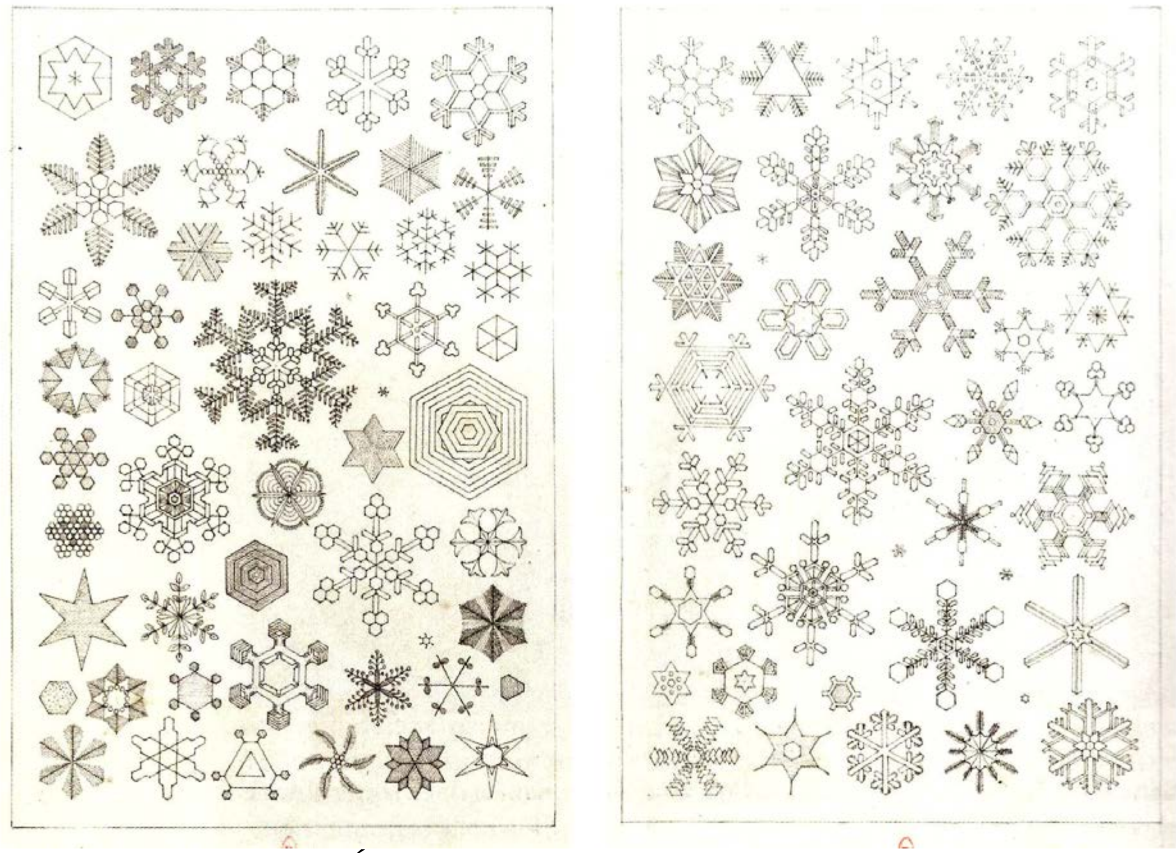

Fig. 12: Jules Bourgoin. «Étoiles de neige». Preparatory plates for an unpublished project of «Revue de l'ornement», (around 1877)

So we can link Bourgoin's studies of snowflakes to his more abstract and systematic studies, based upon combinatorics (which he seriously studied in mathematics books), and geometrical transformations.

There is an important turn that we can observe in natural history during the 20th century, the passing from description to evolution, from static to dynamic. It is not only Darwin's Theory of Evolution that is at stake here, but also the important contribution of D'Arcy Thompson's *On Growth and Form* []. The main idea is that any natural form is the result of a process, and that the description and classification of forms at one point of time does not suffice. There was somewhat the same turn in linguistics with Noam Chomsky's "transformational" or "generative" grammar. The seminal paper by Stiny and Gips introducing shape grammars [] refers explicitly to Chomski, introducing in the domain of artistic productions the same turn from static to dynamic, from constructive to generative.

In conclusion, one important question arises and will need to be seriously addressed: are shape grammars in some way related to those grammars of the 19th century we have talked about? And, more specifically, can we find in Bourgoin's work premisses of generative processes?

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